Design of Inductors and High Frequency Transformers

Inductors store energy, transformers transfer energy. This is the prime difference. The magnetic cores are significantly different for inductors and high frequency transformers: Inductors need an air gap for storing energy, transformers do not. Transformers for flyback converters have to store energy which means they are not a high frequency transformer but they are in fact an inductor with primary and secondary windings. The material of the cores is normally ferrite. In addition to this other materials with high permeability and with a high saturation point are used.

Calculation of Inductors:

An inductor with certain inductance $L$ and certain peak current $\hat{I}$ can be determined by the following calculation:

Inductors should store energy. The stored energy of an inductor is: $W = \frac{1}{2} \hat{L} \hat{I}^2$. This energy is stored as magnetic field energy, within the ferrite core and within the air gap (see Fig.5.1.1). The higher the required stored energy the larger the required core.

- The size of an inductor is approximately proportional to the stored energy.

![Fig. 5.1.1: inductor with its magnetic and mechanical sizes](image)

The field energy in the inductor is:

$$W = \frac{1}{2} \int H \cdot B \, dV = \frac{1}{2} \int_{Fe} H_{Fe} \cdot B_{Fe} \cdot V_{Fe} + \frac{1}{2} \int_{\delta} H_{\delta} \cdot B_{\delta} \cdot V_{\delta}$$

The magnetic field density $\vec{B}$ is continuous and within the air gap and the ferrite is approximately equal, i.e. $\vec{B} \approx B_{Fe} \approx B_{\delta}$. The magnetic field strength $\vec{H}$ is not continuous, within the air gap it is increased by a factor $\mu_r$ compared to that within the ferrite. If this is substituted into equation (1) and considering

$$\vec{B} = \mu_0 \mu_r \cdot \vec{H}, \quad V_{Fe} = l_{Fe} \cdot A \quad \text{and} \quad V_{\delta} = \delta \cdot A$$

this leads to:
\[ W \approx \frac{1}{2} \frac{B^2}{\mu_0} \left( \frac{l_F}{\mu_r + \delta} \right) \cdot A \]

\( \mu_r \) of the ferrite amounts to 1000...4000. It should be noted that the magnetic length of the ferrite is reduced by \( \mu_r \) in the above equation. Therefore it can be seen that the energy is mainly stored within the air gap.

This leads to:

\[ W \approx \frac{1}{2} \frac{B^2 \cdot A \cdot \delta}{\mu_0} \]

- Inductors require an air gap to store energy.

Because the energy is stored within the air gap, an inductor requires a certain volume for the air gap to store a certain amount of energy. The energy is given by \( \frac{1}{2} L \dot{I}^2 \). The core material has a limit for the maximum magnetic flux density \( B \), this limit is about \( B_{\text{max}} = 0.3 \, \text{T} \) for usual ferrite materials. This leads to a minimum required volume \( V_\delta \) of the air gap:

\[
V_\delta = A \cdot \delta \geq \frac{L \dot{I}^2 \cdot \mu_0}{B_{\text{max}}^2} \quad \text{where} \quad B_{\text{max}} = 0.3 \, \text{T}
\]

Knowing the required volume of the air gap, a core can be selected from a databook of ferrite cores.

The number of turns \( N \) can be calculated with help of the magnetic conductance \( A_L \) (often simply called the \( A_L \)-value):

\[ N = \sqrt{\frac{L}{A_L}} \quad A_L : \text{magnetic conductance} \]

The \( A_L \)-value can be verified from the databook of the ferrite cores.

The maximum flux density should not be higher than 0.3 Tesla. The maximum flux density within the ferrite can be calculated using the data of the core datasheet.

\[ B = \frac{L \cdot \dot{I}}{N \cdot A_{\text{min}}} = \frac{N \cdot A_L \cdot \dot{I}}{A_{\text{min}}} \leq 0.3 \, \text{T} \]

\( A_{\text{min}} \): Minimum cross-cut of the core. The flux density has its maximum at \( A_{\text{min}} \). \( A_{\text{min}} \) can be verified from the datasheet.
Calculation of the wire:
The current density $S$ of the wire can be chosen between 2 and 5 A/mm² (depending on the size and the isolation, which determines the heat transport out of the inductor). This leads to the diameter of the wire $d$:

$$d = \sqrt{\frac{4 \cdot I_{RMS}}{\pi \cdot S}} \quad \text{with} \quad S = 2\ldots3\ldots5 \text{ A/mm}^2$$

Calculation of High Frequency Transformers

A high frequency transformers transfer electric power. Its mechanical size depends on the power to be transferred and on the operating frequency. The higher the frequency the smaller the mechanical size. Usually frequencies are from 20 to 100kHz. The material of the core is ferrite.

Databooks for appropriate cores provide information about the possible transfer power for various cores.

The first step to calculate a high frequency transformer is to choose an appropriate core with the help of the databook, the size of the core is dependent on the transfer power and the frequency. The second step is to calculate the number of primary turns. This number determines the magnetic flux density within the core. The number of secondary turns is the ratio of primary to secondary voltage. Following this the diameters of the primary and secondary conductors can be calculated depending on the RMS-values of the currents.

Calculation of the minimum number of primary turns:

Figure 5.2.1: Voltages and currents at a transformer
The voltage $V_1$ at the primary side of the transformers has a rectangle shape. This causes an input current $I_1$, which is the addition of the back transformed secondary current $I_2$ and the magnetising current $I_M$ (see figure 5.2.1). To keep the magnetising current $I_M$ low, a magnetic core without an air gap is used.

The rectangle voltage $V_1$ causes a triangle shape for the magnetising current $I_M$. The magnetising current is approximately independent of the secondary current $I_2$ (see the simple equivalent circuit in figure 5.2.1). The magnifying current is approximately proportional to the magnetic flux or flux density. The input voltage $V_1$ determines the magnetic flux. The physical correlations are given by Faraday's law of induction: $V = N \cdot \frac{d\Phi}{dt}$.

![Figure 5.2.2: input voltage and magnetic flux density at the transformer](image)

For the transformer in figure 5.2.1 follows:

$$\Delta B = \frac{V_1 \cdot T/2}{N_1 \cdot A}$$

- The change $\Delta B$ of flux density depends on the frequency $f = 1/T$ and the number of turns $N_1$. The higher the frequency and the number of turns the lower the change of flux density.

The minimum number of turns $N_{1\text{,min}}$ can be calculated to ensure that a certain change of flux density $\Delta B$ is not exceeded. The saturation flux density of about $\hat{B} = 0.3$ T (which means $\Delta B \approx 0,6$ T) cannot be used in high frequency transformers. In push-pull converters going around the hysteresis loop with every clock would cause unacceptable losses i.e. heat generation. If no further information on core losses and thermal resistance are available, $\Delta B$ should be limited to $\Delta B = 0,3\ldots0,2$ T for operating frequencies from 20 to 100 kHz. The lower $\Delta B$ the lower the core losses.

This leads to a minimum number of turns for $N_1$:

$$N_{1\text{,min}} \geq \frac{V_1 \cdot T/2}{\Delta B \cdot A_{\text{min}}} \quad \text{where} \quad \Delta B \approx 0,2\ldots0,3 \text{ T}$$

$A_{\text{min}}$: minimum cross-section area of the core. This is where the flux density is at a maximum. $A_{\text{min}}$ can be checked from the datasheet.

**Hint:**

In single ended forward converters the core is magnetised into one polarity only. In push-pull converters the core is magnetised alternating into both polarities.
The calculation of the minimum number of turns $N_{1 \text{min}}$ is equal for these different types of switch mode power supplies.

**Calculation of the winding conductors:**
The diameter of the conductors depends on the RMS-value of the current. The current can be calculated with the power.

For the push-pull converter follows:

$$I_{1 \text{RMS}} \approx \frac{P_{\text{out}}}{V_{\text{in}}} \quad \text{and} \quad I_{2 \text{RMS}} = \frac{P_{\text{out}}}{V_{\text{out}}}$$

For the single ended forward converter follows:

$$I_{1 \text{RMS}} \approx \sqrt{2} \frac{P_{\text{out}}}{V_{\text{in}}} \quad \text{and} \quad I_{2 \text{RMS}} = \sqrt{2} \frac{P_{\text{out}}}{V_{\text{out}}}$$

The magnetising current can be neglected in this calculation. The current density can be chosen in a range of 2 to 5 A/mm, depending on the termal resistance of the choke. The cross-section $A_{\text{wire}}$ and the diameter $d_{\text{wire}}$ can be calculated as follows:

$$A_{\text{wire}} = \frac{I}{S} \quad \text{and} \quad d_{\text{wire}} = \frac{I \cdot 4}{S \cdot \pi} \quad \text{where} \quad S = 2 \ldots 3 \ldots 5 \frac{\text{A}}{\text{mm}^2}$$

**Hint:**

If good coupling is important, the primary and secondary winding should be placed on top of each other. Improved coupling is achieved if the windings are interlocked. The coupling is bad in a) good in b) and in c) about four times better than in b).
HINT: The primary number of turns should not be chosen significantly higher than $N_{1\text{min}}$, otherwise the copper losses of the wire would increase needlessly due to the longer conductor.

HINT: For high frequencies and large diameter of the wire the skin effect should be considered. For operating frequencies of more than 20kHz and diameters of more than 1mm litz wire or copper foil should be used.